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Bell's Theorem - A fallacy of the excluded middle.

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Abstract

Albert Einstein never really accepted quantum mechanics. According to Einstein, quantum mechanics is not a complete physical theory. Einstein's dissatisfaction with quantum mechanics intrinsic randomness lead to the famous Einstein, Podolsky and Rosen thought experiment and the Einstein Podolsky Rosen paradox (EPR paradox) which assumes local realism. The Einstein, Podolsky and Rosen thought experiment was the origin of J. S. Bell's publication in 1964. After the publication of Bell's theorem, a variety of experiments were devised to test Bell's inequalities. One of the first experimental tests of Bell's inequality were performed by Freedman and Clauser (1972). Some dramatic violations of Bell's inequality have been reported by so called Bell test experiments. This is taken as empirical evidence against local realism and as positive evidence in favour of quantum mechanics. It is claimed that under local realist theories Bell inequalities cannot be violated. Contrary to this, quantum mechanics predicts that the Bell inequalities will be violated. Who is right, who is wrong? Does quantum mechanics exclude local realism and thus relativity and vice versa? Are there local realist hopes for quantum mechanics besides of Bell's theorem? However, does parameters exist whether they are measured or not, does measurement disturbs the thing being measured. Nonetheless, the time has come to close the book completely on Bell's theorem. The purpose of this publication is to refute Bell's theorem definitely by the proof that

Bell's theorem is a fallacy of the excluded middle.

Key words: Bell's theorem, Logical fallacy, Refutation, Fallacy of the excluded middle

1. Background

Bishop George Berkeley, an immaterialist and an idealist, was born in or near Kilkenny, Ireland on 12 March 1685. Berkeley's famous principle was ***esse* is *percipi*** or to be is to be perceived. In so far, everything that exists either depends for its existence upon a mind or is a mind. George Berkeley (1685-1753), an idealist philosopher, would likely have been positively very amused about Bell's Theorem. Roughly speaking, according to Bell's Theorem, there is no reality separate from its observation. Thus, according to d'Espagnat we should keep the following in mind: "The doctrine that the world is made up of **objects** whose existence is independent of **human consciousness** turns out to be in conflict with **quantum mechanics**." (d'Espagnat 1979, p. 128). It is important to stress that per Bell's theorem, the existence of quantum mechanics objects is dependent on human mind and consciousness. Can there be any misunderstandings about the philosophical background of Bell's theorem? In so far, according to Bell's inequality, telepathy or the "spooky action at a distance" (Einstein) does in fact occur. From this point of view, at first sight, it appears to be reasonable that Bell's inequality can thus be taken as the best proof known, that the positions of immaterialist or idealist philosopher or telepathy as such is scientifically verified.

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2. Material and Methods

Bell's Theorem is based upon some important assumptions:

1. Classical bivalent logic is valid.
2. Hidden variables exist.
3. Hidden variables are local.

Bell's theorem, a correlation experiment, is based on the laws of classical bivalent logic too. In so far, if we can make the proof, that **Bell's theorem is not compatible with the laws of classical bivalent logic**, the same is refuted. Thus, our starting point is classical bivalent logic. From this point of view, it is time to explore the relationship between Bell's theorem and classical bivalent logic in more detail.

2.1. Classical bivalent logic - a brief overview.

There is a long tradition in the history of science and of philosophy to study logic in the terms of the laws of thought. The three classic laws of thought according to Aristotle are the law of identity, the law of contradiction and the law of the excluded middle. Although the exact nature of logic is still a matter of controversy and intense dispute, the laws of logic are mind-independent and nature grounded. Logic investigates and classifies at the end the most basic laws of nature. Is there something that can be more certain than the laws of classical logic? Some of this laws of classical bivalent logic are the following.

2.1.1. Lex identitatis

The law of identity, the first law of nature or **lex identitatis** according to Barukčić (Barukčić 2006, pp. 55-60), states that something like $\mathbf{A_t}$ at a (space) time t is identical only to itself, it is only itself and without anything else, it is the 'purity' as such, it is without the other of itself, it is without any form of a local hidden variable (Barukčić 2006, pp. 55-60) or

$$\mathbf{A_t = A_t.}$$

Theorem 1. Law of identity.

Let

A_t denote something, a Bernoulli random variable, that is either true (=1) or false (=0) at the (space)time t,

t denote the (space)time t,

then

$$(A_t) = (A_t) .$$

Proof.

(A_t)	(A_t)	$(A_t) = (A_t)$	Equation
1	1	true	(1)
0	0	true	(2)

Q. e. d.

In philosophy, "A_t is A_t" is often called a tautology. Only, if it is the fact that $A_t = A_t$ how can A_t have a reference to something else, to a local hidden variable? If something as such is only itself and nothing else, if $A_t = A_t$ then it cannot contain any " local hidden variable", it is only itself and not equally a local hidden variable too. The lex identitatis excludes local hidden variables and can be used for experiments to proof the existence of the same. If $A_t = A_t$ how can A_t change out of itself and without an urge from an other, how can it become an other? If $A_t = A_t$ then any alteration of A_t is associated with subtle problems.

2.1.2. Lex negationis

Lex negationis, the law of negation, the negation of the identity law, is the other side of the lex identitatis. In mathematics and classical logic, negation is an operation on logical values like 0 and 1 that converts true (=1) to false (=0) and false (=0) to true (=1). The following table of Not A_t (also written as $\sim A_t$ or $\neg A_t$) is a proof of the equivalence of **Not A_t = 1 - A_t** .

Theorem 2. Law of negation.

Let

A_t	denote something, a Bernoulli random variable, that is either true (=1) or false (=0) at the (space)time t,
Not A_t	denote the logical negation of A_t that is either true (=1) or false (=0) at the (space)time t,
t	denote the (space)time t,

then

$$(\text{Not } A_t) = 1 - A_t .$$

Proof.

A_t	**(Not A_t)**	**(1 - A_t)**	Equation
1	0	0	(3)
0	1	1	(4)

Q. e. d.

It is important to stress, that **Not A_t = (1 - A_t)**. No matter how the logical negation is notated or symbolised, the logical negation converts either 0 to 1 or 1 to 0, something in its own other. Logical negation is based on a natural process that is able to convert something in its opposite and vice versa. Einstein's relativistic correction is the natural physical background (Barukčić, Causality, 2006, p. 64) of (logical) negation.

Theorem 3.
The logical negation can be defined in terms of algebra.

Theorem 3. Logical negation and algebra.

Let

A_t	denote something that is either true (=1) or false (=0) at the (space)time t,
Not A_t	denote logical negation of A_t that is either true (=1) or false (=0) at the (space)time t,
C_t	denote something other at the (space)time t,
t	denote the (space) time t,

then

$$\mathbf{A_t + (\text{Not } A_t) = 1.}$$

Proof. Equation

$$\mathbf{A_t = A_t} \quad (5)$$

$$A_t - A_t = 0 \quad (6)$$

$$A_t - A_t = C_t - C_t \quad (7)$$

$$C_t + A_t - A_t = C_t \quad (8)$$

$$A_t + C_t - A_t = C_t \quad (9)$$

Set $C_t = 1$ we obtain

$$A_t + 1 - A_t = 1 \quad (10)$$

Recall, that **Not A_t = 1 - A_t** thus we obtain

$$\mathbf{A_t + (\text{Not } A_t) = 1.} \quad (11)$$

Q. e. d.

2.1.3. Lex contradictionis

The law of contradiction (also called the law of non-contradiction) states that it is not possible that one and the same something (**is and equally is not**) at the same (space) time. The law of contradiction can be expressed as:

$$A_t * (\text{Not } A_t) = 0$$

or

$$1 - (A_t * (\text{Not } A_t)) = 1$$

or

$$\text{Not } (A_t \text{ and } (\text{Not } A_t)) = 1$$

or

$$\text{Not } (A_t \wedge (\text{Not } A_t)) = 1.$$

Theorem 4. Law of contradiction.

Let

A_t	denote something that is either true (=1) or false (=0) at the (space)time t,
Not A_t	denote logical negation of A_t that is either true (=1) or false (=0) at the (space)time t,
t	denote the (space)time t,

then

$$\mathbf{1 - (A_t * (\text{Not } A_t)) = 1 .}$$

Proof. Equation

$$\mathbf{A_t = A_t} \quad (12)$$

$$A_t - A_t = 0 \quad (13)$$

Recall that $1^2 = 1$ or $0^2 = 0$. Since **A is either 0 or 1** it is equally true that **$A^2 = A$**. We obtain

$$A_t - (A_t)^2 = 0 \quad (14)$$

$$A_t - (A_t * A_t) = 0 \quad (15)$$

$$A_t * (1 - (A_t)) = 0 \quad (16)$$

Recall, that Not $A_t = 1 - A_t$ thus we obtain

$$\mathbf{A_t * (\text{Not } A_t) = 0.} \quad (17)$$

$$- A_t * (\text{Not } A_t) = - 0 \quad (18)$$

$$1 - (A_t * (\text{Not } A_t)) = 1 \quad (19)$$

Q. e. d.

2.1.4. Law of the excluded middle - Tertium non datur

The law of the excluded middle has a long history in philosophy. The law of the excluded middle, one of the laws of classical bivalent logic, states that something is either true or false, a third point of view, a hidden variable, is not given, a third point of view is impossible. It would be misleading to say that we can tolerate an impossible third point of view and equally respect the law of the excluded middle of classical logic.

Theorem 5. Law of the excluded middle.

Let

A_t	denote something that is either true (=1) or false (=0) at the (space)time t,
Not A_t	denote logical negation of A_t that is either true (=1) or false (=0) at the (space)time t,
t	denote the (space)time t,

then

$$\mathbf{1 - ((1 - A_t)*(1 - Not\ A_t)) = 1.}$$

Proof. Equation

$$\mathbf{A_t = A_t} \qquad (20)$$

$$A_t - A_t = 0 \qquad (21)$$

$$1 + A_t - A_t = 1 \qquad (22)$$

$$A_t + 1 - A_t = 1 \qquad (23)$$

Recall, that Not $A_t = 1 - A_t$ thus we obtain

$$\mathbf{A_t + (Not\ A_t) = 1} \qquad (24)$$

$$A_t + (Not\ A_t) - \mathbf{0} = 1 \qquad (25)$$

According to the law of contradiction, it is true that $(A_t * (Not\ A_t)) = 0$. Thus we obtain

$$\mathbf{A_t + (Not\ A_t) - (A_t * (Not\ A_t)) = 1} \qquad (26)$$

$$0 + A_t + (Not\ A_t) - (A_t * (Not\ A_t)) = 1 \qquad (27)$$

$$1-1 + A_t + (Not\ A_t) - (A_t * (Not\ A_t)) = 1 \qquad (28)$$

$$1 - (1 - A_t - Not\ A_t +(A_t * (Not\ A_t))) = 1 \qquad (29)$$

$$\mathbf{1 - ((1 - A_t) * (1 - (Not\ A_t))) = 1} \qquad (30)$$

Q. e. d.

According to the known long arm of the law of excluded middle the total of **(A_t v (Not A_t)) is true**. The law of the excluded middle does not comment on what truth values A_t itself in bivalent logic may take. Recall, there are certain systems of logic that reject bivalence. Some of this systems of logic allow more than two truth values. In ternary logic, something may be true, false or unknown. In fuzzy logic something may be true, false or somewhere in between. Other systems of logic do not accept the law of excluded middle. The law of the excluded middle in bivalent logic is commonly referred to as a false dilemma, the fallacy of the excluded middle.

Theorem 6. Law of the excluded middle and algebra I.

Let

A_t	denote something that is either true (=1) or false (=0) at the (space) time t,
Not A_t	denote logical negation of A_t which is either true (=1) or false (=0) at the (space) time t,
t	denote the (space)time t,

then

$$(A_t \text{ v } (\text{Not } A_t)) = (1- ((1 - A_t) * (1 - (\text{Not } A_t)))) = 1.$$

Proof.

A_t	Not A_t	**(A_t v (Not A_t))**	(1-A_t)	(1-(Not A_t))	**(1- ((1 -A_t) * (1-(Not A_t))))**	**Eq.**
1	0	**1**	0	1	(1 - ((0) * (1)))= **1**	(31)
0	1	**1**	1	0	(1 - ((1) * (0)))= **1**	(32)

Q. e. d.

The identity, the equivalence of

$$(A_t \text{ v } (\text{Not } A_t)) = 1 = (1 - ((1 - A_t) * (1 - (\text{Not } A_t))))$$

is proofed and correct at least for bivalent logic. In so far, we are able to express the law of the excluded middle algebraically.

Theorem 7. Law of the excluded middle and algebra II.

Let

A_t	denote something that is either true (=1) or false (=0) at the (space) time t,
Not A_t	denote logical negation of A_t which is either true (=1) or false (=0) at the (space) time t,
t	denote the (space)time t,

then

$$(A_t \text{ v } (\text{Not } A_t)) = (A_t) + (\text{Not } A_t) - ((A_t)* (\text{Not } A_t)) = 1.$$

Proof.

A_t	Not A_t	((A_t)* (Not A_t))	**A_t + (Not A_t) - ((A_t)* (Not A_t))**	**(A_t v (Not A_t))**		**Eq.**
1	0	**0**	**1**	**1**	**True**	(33)
0	1	**0**	**1**	**1**	**True**	(34)

Q. e. d.

The identity, the equivalence of

$$(A_t \text{ v } (\text{Not } A_t)) = 1 = (A_t) + (\text{Not } A_t) - ((A_t)*(\text{Not } A_t))$$

is proofed correct at least under the conditions of classical logic. This form of the law of the excluded middle is very important for the proof of Bell' theorem. But we need more then this. We need to express the law of the excluded middle in terms of an inequality.

Theorem 8. Law of the excluded middle in terms of an inequality.

Let

A_t	denote something that is either true (=1) or false (=0) at the (space) time t,
Not A_t	denote logical negation of A_t which is either true (=1) or false (=0) at the (space) time t,
t	denote the (space)time t,

then

$$\mathbf{(A_t) + (Not\ A_t) - ((A_t)^* (Not\ A_t)) \geq 0.}$$

Proof.

A_t	Not A_t	**(A_t v (Not A_t))**	**A_t + (Not A_t) - ((A_t)* (Not A_t))**			Equation
1	0	**1**	**1**	**≥ 0**	**True**	(35)
0	1	**1**	**1**	**≥ 0**	**True**	(36)

Q. e. d.

The inequality

$$\mathbf{(A_t) + (Not\ A_t) - ((A_t)^*(Not\ A_t)) \geq 0}$$

is proofed as mathematically correct and is of course valid for ever. This form of the law of the excluded middle is very important for the proof of Bell' theorem. A relationship between a A_t and an other variable C_t is possible. Thus, we obtain another inequality.

Theorem 9. The inequality of A_t and C_t.

Let

A_t	denote the random variable A_t which is either true (=1) or false (=0) at the (space) time t,
Not A_t	denote logical negation of the random variable A_t which is either true (=1) or false (=0) at the (space) time t,
C_t	denote the random variable C_t which is either true (=1) or false (=0) at the (space) time t,
Not C_t	denote logical negation of the random variable C_t which is either true (=1) or false (=0) at the (space) time t,
t	denote the (space)time t,

then

$$\mathbf{((A_t)^* (Not\ C_t)) + ((Not\ A_t)^* (C_t)) \geq 0.}$$

Proof.

A_t	Not A_t	C_t	Not C_t	((A_t)*(Not C_t))	((Not A_t)*(C_t))	((A_t)*(Not C_t)) + ((Not A_t)*(C_t))		**Eq.**
1	0	1	0	0	0	**0**	**≥ 0**	(37)
1	0	0	1	1	0	**1**	**≥ 0**	(38)
0	1	1	0	0	1	**1**	**≥ 0**	(39)
0	1	0	1	0	0	**0**	**≥ 0**	(40)

Q. e. d.

In general, it is accepted, that $0 \geq 0$. Thus, the equation $\mathbf{((A_t)^*(Not\ C_t)) + ((Not\ A_t)^* (C_t)) \geq 0}$ is proofed as correct. The inequality $((A_t)^*(Not\ C_t) + ((Not\ A_t)^*(C_t))) \geq 0$ is based on the **either** A **or** C relationship between A and C. Now suppose that we want to determine whether there are local hidden variables, the law of the excluded middle could be used for this purposes. Observe, however, that the law of excluded middle can be misapplied and leading thus to the logical fallacy of the excluded middle, also known as a false dilemma. Can we turn back time prior to 1964? Thus, let us go all the way to the end.

2.1.5. Law of the excluded middle and local hidden variables

Theorem 10. The law of the excluded middle and local hidden variable.

Let

A_t	denote the Bernoulli random variable A_t is either true (=1) or false (=0) at the (space)time t,
Not A_t	denote the logical negation of the Bernoulli random variable A_t that is either true (=1) or false (=0) at the (space) time t,
B_t	denote the Bernoulli random variable B_t that is either true (=1) or false (=0) at the (space)time t,
Not B_t	denote the logical negation of the Bernoulli random variable B_t that is either true (=1) or false (=0) at the (space) time t,
C_t	denote the local hidden variable which is either true (=1) or false (=0) at the (space)time t,
Not C_t	denote the logical negation of the local hidden variable which is either true (=1) or false (=0) at the (space)time t,
t	denote the (space)time t,

then

$$\mathbf{(A_t * (Not\ C_t)) = (Not\ C_t) - ((B_t) * (Not\ C_t)).}$$

Proof. Equation

$$\mathbf{A_t = A_t} \quad (41)$$

$$A_t - A_t = 0 \quad (42)$$

$$1 + A_t - A_t = 1 \quad (43)$$

$$A_t + 1 - A_t = 1 \quad (44)$$

Recall, that Not $A_t = 1 - A_t$ thus we obtain

$$\mathbf{A_t + (Not\ A_t) = 1} \quad (45)$$

$$A_t + (Not\ A_t) - \mathbf{0} = 1 \quad (46)$$

According to the law of contradiction, it is true that $\mathbf{(A_t * (Not\ A_t)) = 0}$.
Thus we obtain

$$A_t + (Not\ A_t) - (A_t * (Not\ A_t)) = 1 \quad (47)$$

$$0 + A_t + (Not\ A_t) - (A_t * (Not\ A_t)) = 1 \quad (48)$$

$$1-1 + A_t + (Not\ A_t) - (A_t * (Not\ A_t)) = 1 \quad (49)$$

$$1 - (1 - A_t - Not\ A_t + (A_t * (Not\ A_t))) = 1 \quad (50)$$

$$\mathbf{1 - ((1 - A_t) * (1 - (Not\ A_t))) = 1} \quad (51)$$

$$(Not\ C_t) * (1-((1-A_t)*(1-(Not\ A_t)))) = (Not\ C_t) \quad (52)$$

$$(Not\ C_t) * (A_t + (Not\ A_t)) = (Not\ C_t) \quad (53)$$

$$(A_t*(Not\ C_t))+((Not\ A_t)*(Not\ C_t)) = (Not\ C_t) \quad (54)$$

$$(A_t*(Not\ C_t)) = (Not\ C_t) - ((Not\ A_t)*(Not\ C_t)) \quad (55)$$

Set $\mathbf{B_t = (Not\ A_t)}$. We obtain

$$\mathbf{(A_t * (Not\ C_t)) = (Not\ C_t) - ((B_t) * (Not\ C_t)).} \quad (56)$$

Q. e. d.

In so far, even if there is "a third", a local hidden variable, in relation to A_t and B_t, where $\mathbf{B_t = (Not\ A_t)}$, it must hold true that $\mathbf{A_t * (Not\ C_t) = (Not\ C_t) - (B_t) * (Not\ C_t)}$.

2.2. Bell's thought experiment

The system considered by Bell (Wigner 1970, p. 1005) is based on an "example advocated by Bohm and Aharonov" (Bell 1964, p. 195). Bell is considering "a pair of spin one-half particles formed somehow in the singlet spin state and moving freely in opposite directions. Measurements can be made, say by Stern-Gerlach magnets, on selected components of the spins $\vec{\sigma}_1$, and $\vec{\sigma}_2$. If measurement of the components $\vec{\sigma}_1 * \vec{a}$, where $\vec{a}$ is some unit vector, yields the value + 1 then, according to quantum mechanics, measurement of $\vec{\sigma}_2 * \vec{a}$ must yield the value -1 and vice versa. Now ... it seems one at least worth considering, that if the two measurements are made at places remote from one another the orientation of one magnet does not influence the result obtained with the other. Since we can predict in advance the result of measuring any chosen component of $\vec{\sigma}_2$, by previously measuring the same component of $\vec{\sigma}_1$, it follows that the result of any such measurement must actually be predetermined. Since the initial quantum mechanical wave function does not determine the result of an individual measurement, this predetermination implies the possibility of a more complete specification of the state.

Let this more complete specification be effected by means of parameters λ. It is a matter of indifference in the following whether λ denotes a single variable or a set, or even a set of functions, and whether the variables are discrete or continuous." (Bell 1964, p 195).

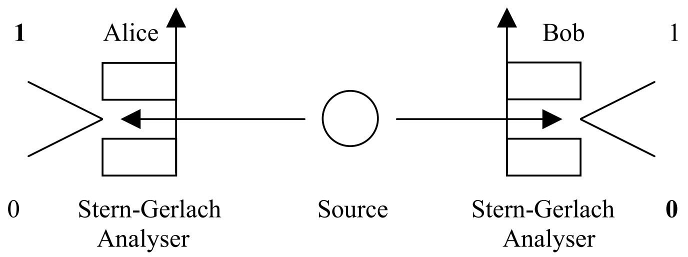

Figure. Bell`s **thought** experiment.

2.3. d'Espagnat's proof

J. S. Bell published his theorem in the year 1964 (Bell 1964), other versions of Bell's theorem are meanwhile published too. Here we will use a form of Bell's inequality which is based on the well known d'Espagnat's proof (d'Espagnat 1979). The proof by d'Espagnat (Harrison 1982, p. 812) offer the possibility to analyse Bell's theorem by the tools of classical bivalent logic. Bell himself advocated (Bell 1981, pp. C2 49-52) d'Espagnat's proof (Harrison 1982, p. 812) too. According to d'Espagnat, Bell's "inequality applies to experiments with particles that have three stable properties A, B and C, each of which can have the values plus and minus." (d'Espagnat 1979, p. 132). However, the Wigner-d'Espagnat inequality "trivial as it is, the inequality is not respected by quantum mechanical probabilities." (Bell 1981, p. C2 52). According to the known d'Espagnat's proof we can rewrite Bell's theorem more compactly (Harrison 1982, p. 812) in the short form as

$$\sum_{t=1}^{n}\left(A_t * \left(\text{Not } B \right)_t \right) + \sum_{t=1}^{n}\left(B_t * \left(\text{Not } C \right)_t \right) \geq \sum_{t=1}^{n}\left(A_t * \left(\text{Not } C \right)_t \right) \qquad (57)$$

Roughly speaking, this is the so called *Bell's inequality* in terms of classical logic. Nonetheless, some people have trouble with Bell's theorem and rightly too. At this point, however, it is important to explore Bell's theorem from the standpoint of classical logic.

2.4. Bell's theorem and classical logic

It is important to derive a logical function from Eq. (57) to be able to analyse Bell's theorem in terms of classical logic.

Theorem 11. Bell's inequality in terms of classical logic.

Let

A_t	denote the Bernoulli random variable A_t which is either true (=1) or false (=0) at the (space) time t,
Not A_t	denote the logical negation of the Bernoulli random variable A_t that is either true (=1) or false (=0) at the (space) time t,
B_t	denote the Bernoulli random variable B_t that is either true (=1) or false (=0) at the (space)time t,
Not B_t	denote the logical negation of the Bernoulli random variable B_t that is either true (=1) or false (=0) at the (space) time t,
C_t	denote the local hidden variable which is either true (=1) or false (=0) at the (space)time t,
Not C_t	denote the logical negation of the local hidden variable which is either true (=1) or false (=0) at the (space) time t,
t	denote the (space) time t,

then

$$(A_t * (\text{Not } B)_t) + (B_t , * (\text{Not } C)_t) \geq (A_t *(\text{Not } C)_t).$$

Proof. **Equation**

$$\sum_{t=1}^{n}\left(A_t * \left(\text{Not } B \right)_t \right) + \sum_{t=1}^{n}\left(B_t * \left(\text{Not } C \right)_t \right) \geq \sum_{t=1}^{n}\left(A_t * \left(\text{Not } C \right)_t \right) \qquad (58)$$

Bell's inequality is correct and valid at every single measurement. W e set **n = 1**.

$$\sum_{t=1}^{1}\left(A_t * \left(\text{Not } B \right)_t \right) + \sum_{t=1}^{1}\left(B_t * \left(\text{Not } C \right)_t \right) \geq \sum_{t=1}^{1}\left(A_t * \left(\text{Not } C \right)_t \right) \qquad (59)$$

$$(A_t *(\text{Not } B)_t) + (B_t *(\text{Not } C)_t) \geq (A_t *(\text{Not } C)_t) \qquad (60)$$

Q. e. d.

In so far, at the end, Bell's theorem can be expressed in terms of a logical inequality as

$$(A_t *(\text{Not } B)_t) + (B_t * (\text{Not } C)_t) \geq (A_t *(\text{Not } C)_t). \qquad (61)$$

This inequality can be rewritten as

$$(A_t , (\text{Not } C)_t) \leq (A_t , (\text{Not } B)_t) + (B_t , (\text{Not } C)_t).$$

3. Results

A great deal of this work is to discover whether Bell's theorem is valid and correct. But how can we know, how can we proof that Bell's reasoning is correct? In so far, how do we start? The theoretical and experimental investigations concerning Bell's theorem are based on some assumptions. First of all, according to Bell, classical logic is valid and thus the law of the excluded middle. Is there a relationship between the law of the excluded middle and Bell's theorem? If yes, how does Bell define the law of the excluded middle? In so far, we start our proof of Bell's theorem with classical logic or in more detail with the law of the excluded middle.

Theorem 12. Bell's theorem - a logical fallacy of the excluded middle.

Let

A_t	denote the random variable A_t which is either true (=1) or false (=0) at the (space) time t,
Not A_t	denote the logical negation of the random variable A_t which is either true (=1) or false (=0) at the (space) time t,
B_t	denote the random variable B_t which is either true (=1) or false (=0) at the (space) time t,
Not B_t	denote the logical negation of the random variable B_t which is either true (=1) or false (=0) at the (space) time t,
C_t	denote the random variable C_t which is either true (=1) or false (=0) at the (space) time t,
Not C_t	denote the logical negation of the random variable C_t which is either true (=1) or false (=0) at the (space) time t,
t	denote the (space)time t. Bell's theorem is based on the inequality

$$\mathbf{(1 - ((1 - (A_t))*(1 - (\text{Not } A_t)))) \geq (\text{Not } A_t) + ((\text{Not } C_t)*((A_t) - (B_t))).}$$

Proof. Equation

$$\sum_{t=1}^{n}\left(A_t * \left(\text{NotB}\right)_t\right) + \sum_{t=1}^{n}\left(B_t * \left(\text{NotC}\right)_t\right) \geq \sum_{t=1}^{n}\left(A_t * \left(\text{NotC}\right)_t\right) \quad (62)$$

Bell's inequality is correct and valid at every single measurement. We set **n = 1**.

$$\sum_{t=1}^{1}\left(A_t * \left(\text{NotB}\right)_t\right) + \sum_{t=1}^{1}\left(B_t * \left(\text{NotC}\right)_t\right) \geq \sum_{t=1}^{1}\left(A_t * \left(\text{NotC}\right)_t\right) \quad (63)$$

Eq. 63 is identical with Eq. 64.

$$\mathbf{((A_t) * (\text{Not } B_t)) + ((B_t) * (\text{Not } C_t)) \geq ((A_t)*(\text{Not } C_t))} \quad | + ((A_t)*(B_t)) \quad (64)$$

$$((A_t)*(\text{Not } B_t)) + \mathbf{((A_t)*(B_t))} + ((B_t)*(\text{Not } C_t)) \geq ((A_t)*(\text{Not } C_t)) + \mathbf{((A_t)*(B_t))} \quad (65)$$

$$(A_t)*((B_t) + (\text{Not } B_t)) + ((B_t)*(\text{Not } C_t)) \geq ((A_t)*(\text{Not } C_t)) + ((A_t)*(B_t)) \quad (66)$$

$$(A_t)*(1) + ((B_t)*(\text{Not } C_t)) \geq ((A_t)*(\text{Not } C_t)) + ((A_t)*(B_t)) \quad (67)$$

$$(A_t) + ((B_t)*(\text{Not } C_t)) \geq ((A_t)*(\text{Not } C_t)) + ((A_t)*(B_t)) \quad | - ((B_t)*(\text{Not } C_t)) \quad (68)$$

$$(A_t) \geq ((A_t)*(\text{Not } C_t)) + ((A_t)*(B_t)) - \mathbf{((B_t)*(\text{Not } C_t))} \quad | + (\text{Not } A_t) \quad (69)$$

$$(A_t) + \mathbf{(\text{Not } A_t)} \geq ((A_t)*(\text{Not } C_t)) + ((A_t)*(B_t)) + \mathbf{(\text{Not } A_t)} - ((B_t)*(\text{Not } C_t)) \quad | - ((A_t)*(\text{Not } A_t)) \quad (70)$$

$$(A_t) + (\text{Not } A_t) - ((A_t)*(\text{Not } A_t)) \geq ((A_t)*(\text{Not } C_t)) + (\text{Not } A_t) - ((B_t)*(\text{Not } C_t)) - ((A_t)*(\text{Not } A_t)) \quad | +0 \tag{71}$$

$$0 + (A_t) + (\text{Not } A_t) - ((A_t)*(\text{Not } A_t)) \geq ((A_t)*(\text{Not } C_t)) + (\text{Not } A_t) - ((B_t)*(\text{Not } C_t)) - ((A_t)*(\text{Not } A_t)) + 0 \quad | \ 0 = +1 - 1 \tag{72}$$

$$1 - 1 + (A_t) + (\text{Not } A_t) - ((A_t)*(\text{Not } A_t)) \geq ((A_t)*(\text{Not } C_t)) + (\text{Not } A_t) - ((B_t)*(\text{Not } C_t)) - ((A_t)*(\text{Not } A_t)) \tag{73}$$

Recall, according to Eq. (17) it is $((A_t)*(\text{Not } A_t)) = 0$.

$$(1 - ((1 - (A_t))*(1 - (\text{Not } A_t)))) \geq ((A_t)*(\text{Not } C_t)) + (\text{Not } A_t) - ((B_t)*(\text{Not } C_t)) - 0 \tag{74}$$

The term $(1 - ((1 - (A_t))*(1 - (\text{Not } A_t))))$ is known to be identical with the definition of the law of the excluded middle as proofed in Eq. (30), (31), (32), (33) and (34). In so far, Bell's inequality is based on the law of the excluded middle and thus on classical logic. It is possible to derive the pure law of the excluded middle from Bell's inequality. This is very important.

$$(1 - ((1 - (A_t))*(1 - (\text{Not } A_t)))) \geq (\text{Not } A_t) + (\text{Not } C_t)*((A_t) - (B_t)) \tag{75}$$

Q. e. d.

In so far, contrary to expectation, Bell is right, Bell's theorem is indeed based on classical logic, that is to say on the law on the excluded middle. We started with Bell's inequality (Eq. 62) and were able to derive the pure law of the excluded middle. This can be seen on the left side of the Eq. (75). The law of the excluded middle in the form of $(1 - ((1 - (A_t))*(1 - (\text{Not } A_t)))) = 1$ was already proofed as correct (Eq. (30), (31), (32), (33), (34)). In so far, Bell's theorem is indeed based on classical logic that is to say on the law on the excluded middle. Bell is claiming to respect classical logic and rightly too. Eq. (75) is the proof that Bell's theorem is based on classical bivalent logic. In so far, until Eq. (75), I don't see a possibility to contradict Bell. Is it possible that the Eq. (75) which states

$$(1 - ((1 - (A_t))*(1 - (\text{Not } A_t)))) \geq (\text{Not } A_t) + (\text{Not } C_t)*((A_t) - (B_t))$$

is not valid. J. S. Bell is assuming "a pair of spin one-half particles formed somehow in the singlet spin state and **moving freely in opposite** directions" (Bell 1964, p 195). This is respected by the variables A, B and C. Our central idea, Eq. (75), the starting point of our refutation of Bell's theorem, as derived above and expressed according to Eq. (75) could be mathematically not correct. The construction of truth-tables is a simple, useful and reliable method of evaluating the validity of arguments. Thus let us proof Eq. (75).

Theorem 13. The inequality of the Equation 75 is correct.

Let

A_t	denote the random variable A_t which is either true (=1) or false (=0) at the (space) time t,
Not A_t	denote logical negation of the random variable A_t which is either true (=1) or false (=0) at the (space) time t,
C_t	denote the random variable C_t which is either true (=1) or false (=0) at the (space) time t,
Not C_t	denote logical negation of the random variable C_t which is either true (=1) or false (=0) at the (space) time t,
t	denote the (space)time t,

then it is true that

$$(1 - ((1 - (A_t))*(1 - (\text{Not } A_t)))) \geq (\text{Not } A_t) + (\text{Not } C_t)*((A_t) - (B_t)).$$

Proof.

A_t	B_t	C_t	Not C_t	$((A_t)-(B_t))$	(Not C_t)*((A_t)-(B_t))	(1-((1-(A_t))*(1-(Not A_t))))	(Not A_t) +(Not C_t)*((A_t)-(B_t))		Eq.
						(1)	**(2)**	(1) ≥(2)	
1	1	1	0	0	0	**1**	**0**	**True**	(76)

In general, it is said that (a ≥ b) is true if it is equally true that ((a = b) = true) or ((a > b) = true). In this case, the inequality (1- ((1- (A_t))*(1- (Not A_t)))) ≥ 0 is mathematically true, because **1 > 0**.

1	1	0	1	0	0	**1**	**0**	**True**	(77)
1	0	1	0	1	0	**1**	**0**	**True**	(78)
1	0	0	1	1	1	**1**	**1**	**True**	(79)

In general, it is said that (a ≥ b) is true if it is equally true that ((a = b) = true) or ((a > b) = true). In this case, the inequality (1- ((1- (A_t))*(1- (Not A_t)))) ≥ 1 is mathematically true, because **1 = 1**.

0	1	1	0	-1	- 0	**1**	**1**	**True**	(80)
0	1	0	1	-1	-1	**1**	**0**	**True**	(81)
0	0	1	0	0	0	**1**	**1**	**True**	(82)
0	0	0	1	0	0	**1**	**1**	**True**	(83)

Q. e. d.

Equation (75) is proofed as mathematically correct. Is there still hope to refute Bell's theorem, does it make sense to move on, must we accept Bell's theorem as correct, definitely? The central idea, the starting point of Bell's inequality as proofed above can be expressed according to Eq. (75) formally as

$$\mathbf{(1 - ((1 - (A_t))* (1 - (Not\ A_t)))) \geq (Not\ A_t) + (Not\ C_t) * ((A_t) - (B_t)).}$$

This equation is proofed as mathematically correct! Is there still an error?

Bell's Theorem - Proof by contradiction

Before preceding to the definite refutation of Bell's theorem, it is useful to address some preliminary points. Further, we will use the technique of the proof by contradiction to proof whether Bell`s theorem is true or not, which mathematicians use extensively. How does a proof by contradiction works? We wish to proof that Bell's theorem is false. We use the proof by contradiction for our purposes by assuming the opposite of that what we want to proof. In so far, we assume that **Bell's theorem is**, in fact, **true**. Based on this assumption we try to see what conclusions can be drawn out from a Bell's theorem that is assumed to be true. We try to derive a contradiction from this starting point (something that disagrees with logic or mathematics that is already known to be true). If our assumption, that Bell's theorem is true, leads to a **logical contradiction** then we must conclude after all that our assumption, that Bell's theorem is true, must be false. In so far, our proof would be complete. Let us refute now very precisely Bell`s theorem.

Bell's theorem - reductio ad absurdum I.

Theorem 14. Bell's theorem contradicts classical logic and leads to a logical contradiction - reductio ad absurdum I.

Premises.

Let

A_t	denote the random variable A_t which is either true (=1) or false (=0) at the (space) time t,
Not A_t	denote logical negation of the random variable A_t which is either true (=1) or false (=0) at the (space) time t,
C_t	denote the random variable C_t which is either true (=1) or false (=0) at the (space) time t,
Not C_t	denote logical negation of the random variable C_t which is either true (=1) or false (=0) at the (space) time t,
t	denote the (space)time t.

Let Bell's inequality be denoted by

$$\mathbf{(1 - ((1 - (A_t))*(1 - (Not\ A_t)))) \geq (Not\ A_t) + (Not\ C_t)*((A_t) - (B_t)).}$$

Assumption.

We want to prove that Bell's theorem is false. Thus, let us assume the opposite, the logical negation, of that what we want to proof. Let us assume that **Bell's theorem is correct**, that is to say true.

Proof by contradiction.

Eq.

$$(1 - ((1 - (A_t))*(1 - (Not\ A_t)))) \geq ((Not\ A_t) + (Not\ C_t)*((A_t) - (B_t))) \tag{84}$$

The term $((Not\ A_t) + (Not\ C_t)*((A_t) - (B_t)))$ can take the values 0 or 1 according to Eq. (76) - (83). In so far, let us assume, that $\mathbf{((Not\ A_t) + (Not\ C_t)*((A_t) - (B_t))) = 0}$. We obtain Eq. (85).

$$(1 - ((1 - (A_t))*(1 - (Not\ A_t)))) \geq (((Not\ A_t) + (Not\ C_t)*((A_t) - (B_t))) = \mathbf{0}). \tag{85}$$

It is generally accepted, that ($a \geq b$) means that ($a = b$) **or** ($a > b$), both are equally allowed and possible, if the inequality is true. In so far, Eq. (84) is true, if $((1-((1-(A_t))*(1-(Not\ A_t)))) = 0)$. Eq. (84) is equally true if $((1-((1-(A_t))*(1-(Not\ A_t)))) > 0)$. In this case, let us assume, that

$$(1-((1-(A_t))*(1-(Not\ A_t)))) = (((Not\ A_t) + (Not\ C_t)*((A_t) - (B_t))) = 0),$$

which satisfies Bell's inequality. On the other hand, Bell is respecting classical logic and thus the law of the excluded middle as proofed in Eq. (75). The law of excluded middle in classical bivalent logic must yield $\mathbf{(1-((1-(A_t))*(1-(Not\ A_t)))) = 1}$. Bell's inequality is respecting this law. We obtain Eq. (86).

$$((1 - ((1 - (A_t))*(1 - (Not\ A_t)))) = \mathbf{1}) = (((Not\ A_t) + (Not\ C_t)*((A_t) - (B_t))) = 0) \tag{86}$$

Eq. (86) is equal to Eq. (87).

$$\mathbf{1 = 0.} \tag{87}$$

Bell's inequality leads to a logical contradiction, it not true that 1 = 0. Therefore, our original assumption, that Bell's theorem is correct is false.

Conclusion.
Bell's theorem contradicts classical logic and leads to a logical contradiction!
Bell's theorem is refuted!
Q. e. d.

There is more then an overwhelming evidence that Bell's theorem is not correct. In general, it is said that (a ≥ b) is true if ((a = b) = true) **or** ((a > b) = true). In so far, the inequality (a ≥ b) as such is respected if ((a = b) = true) or if ((a > b) = true) both is allowed, possible and correct. In both cases, the law of the excluded middle must be respected and not only in one of them. It is worth to mention, that mathematically it is true that 1 ≥ 0. Bell`s inequality must at the same (space) time respect the law of the excluded middle to. And this is not the fact. If $(1-((1-(A_t))*(1-(\text{Not } A_t)))) = 0$, then Bell is not respecting classical logic but besides of this, his inequality is correct. His is claiming to respect classical logic but in fact he is not doing that. However, sometimes it is often the best to make a proof more. In so far, our long and painful process of discovering whether Bell's theorem is correct or not has still not reached the end. Since Bell`s theorem is misleading, wrongly focused and badly pronounced it could be the effort worth.

Bell's theorem - reductio ad absurdum II.

Theorem 15. Bell's theorem contradicts classical logic and leads to a logical contradiction - reductio ad absurdum II.

Premises.

Let

A_t	denote the random variable A_t which is either true (=1) or false (=0) at the (space) time t,
Not A_t	denote logical negation of the random variable A_t which is either true (=1) or false (=0) at the (space) time t,
C_t	denote the random variable C_t which is either true (=1) or false (=0) at the (space) time t,
Not C_t	denote logical negation of the random variable C_t which is either true (=1) or false (=0) at the (space) time t,
t	denote the (space)time t.

Let Bell's inequality be denoted by

$$\mathbf{(1 - ((1 - (A_t))* (1 - (\text{Not } A_t)))) \ \geq (\text{Not } A_t) + (\text{Not } C_t) * ((A_t) - (B_t)).}$$

Assumption.

We want to prove that Bell's theorem is false. Thus, let us assume the opposite, the logical negation, of that what we want to proof. Let us assume that **Bell's theorem is correct**, that is to say true.

Proof by contradiction.

Eq.

$$(1 - ((1 - (A_t))* (1 - (\text{Not } A_t)))) \ \geq ((\text{Not } A_t) + (\text{Not } C_t) * ((A_t) - (B_t))) \qquad (88)$$

The term $((\text{Not } A_t) + (\text{Not } C_t) * ((A_t) - (B_t)))$ can take the values 0 or 1 according to Eq. (76) - (83). In so far, let us assume, that $\mathbf{((Not\ A_t) + (Not\ C_t) * ((A_t) - (B_t))) = 0}$. We obtain Eq. (89).

$$(1 - ((1 - (A_t))* (1 - (\text{Not } A_t)))) \geq (((\text{Not } A_t) + (\text{Not } C_t) * ((A_t) - (B_t))) = \mathbf{0}). \qquad (89)$$

It is generally accepted, that (a ≥ b) means that (a = b) **or** (a > b), both are equally allowed and possible, if the inequality is true. In so far, Eq. (88) is true, if $((1-((1-(A_t))*(1-(\text{Not } A_t)))) = 0)$. Eq. (88) is equally true if $((1-((1-(A_t))*(1-(\text{Not } A_t)))) > 0)$. In this case, let us assume, that

$$(1-((1-(A_t))*(1-(\text{Not } A_t)))) > (((\text{Not } A_t) + (\text{Not } C_t) * ((A_t) - (B_t))) = 0),$$

which satisfies Bell's inequality. On the other hand, Bell is respecting classical logic and thus the law of the excluded middle as proofed in Eq. (75). The law of excluded middle in classical bivalent logic must yield $\mathbf{(1-((1-(A_t))*(1-(Not\ A_t)))) = 1}$. Bell's inequality is respecting this law. We obtain Eq. (90).

$$((1 - ((1 - (A_t))* (1 - (\text{Not } A_t)))) = \mathbf{1}) > (((\text{Not } A_t) + (\text{Not } C_t) * ((A_t) - (B_t))) = 0) \qquad (90)$$

Eq. (90) is equal to Eq. (91).

$$\mathbf{1 > 0.} \qquad (91)$$

It true that 1 > 0, only according to Bell's inequality, the law of the excluded middle is respected if $\mathbf{(1-((1-(A_t))*(1-(Not\ A_t)))) > 0}$, which is not what classical logic demands. Classical logic demands that the law of the excluded middle is respected if $\mathbf{(1-((1-(A_t))*(1-(Not\ A_t))))=1.}$ Bell is claiming to respect classical logic, but in fact he does not. Therefore, Bell's inequality leads once again to a logical contradiction. Our original assumption, that Bell's theorem is correct, is false.

Conclusion.

Bell's theorem contradicts classical logic and leads to a logical contradiction!

Bell's theorem is refuted!

Q. e. d.

The part $((\text{Not } A_t) + (\text{Not } C_t) * ((A_t) - (B_t)))$ of 's inequality can take the value 1. This should be investigated too.

Bell's theorem - reductio ad absurdum III.

Theorem 16. Bell's theorem contradicts classical logic and leads to a logical contradiction - reductio ad absurdum III.

Premises.

Let

A_t	denote the random variable A_t which is either true (=1) or false (=0) at the (space) time t,
Not A_t	denote logical negation of the random variable A_t which is either true (=1) or false (=0) at the (space) time t,
C_t	denote the random variable C_t which is either true (=1) or false (=0) at the (space) time t,
Not C_t	denote logical negation of the random variable C_t which is either true (=1) or false (=0) at the (space) time t,
t	denote the (space)time t.

Let Bell's inequality be denoted by

$$\mathbf{(1 - ((1 - (A_t))^* (1 - (Not\ A_t)))) \geq (Not\ A_t) + (Not\ C_t) * ((A_t) - (B_t)).}$$

Assumption.

We want to prove that Bell's theorem is false. Thus, let us assume the opposite, the logical negation, of that what we want to proof. Let us assume that **Bell's theorem is correct**, that is to say true.

Proof by contradiction.

Eq.

$$(1 - ((1 - (A_t))^* (1 - (Not\ A_t)))) \geq ((Not\ A_t) + (Not\ C_t) * ((A_t) - (B_t))) \tag{92}$$

The term $((Not\ A_t) + (Not\ C_t) * ((A_t) - (B_t)))$ can take the values 0 or 1 according to Eq. (76) - (83). In so far, let us assume, that $\mathbf{((Not\ A_t) + (Not\ C_t) * ((A_t) - (B_t))) = 1}$. We obtain Eq. (93).

$$(1 - ((1 - (A_t))^* (1 - (Not\ A_t)))) \geq (((Not\ A_t) + (Not\ C_t) * ((A_t) - (B_t))) = \mathbf{1}). \tag{93}$$

It is generally accepted, that (a ≥ b) means that (a = b) **or** (a > b), both are equally allowed and possible, if the inequality is true. In so far, Eq. (92) is true, if $((1-((1-(A_t))^*(1-(Not\ A_t)))) \mathbf{=} 0)$. Eq. (92) is equally true if $((1-((1-(A_t))^*(1-(Not\ A_t)))) > 0)$. In this case, let us assume, that

$$(1-((1-(A_t))^*(1-(Not\ A_t)))) \mathbf{=} (((Not\ A_t) + (Not\ C_t) * ((A_t) - (B_t))) = 0),$$

which satisfies Bell's inequality. On the other hand, Bell is respecting classical logic and thus the law of the excluded middle as proofed in Eq. (75). The law of excluded middle in classical bivalent logic must yield $\mathbf{(1-((1-(A_t))^*(1-(Not\ A_t)))) = 1}$. Bell's inequality is respecting this law. We obtain Eq. (94).

$$((1 - ((1 - (A_t))^* (1 - (Not\ A_t)))) = \mathbf{1}) = (((Not\ A_t) + (Not\ C_t) * ((A_t) - (B_t))) = 1) \tag{94}$$

Eq. (94) is equal to Eq. (95).

$$\mathbf{1 = 1.} \tag{95}$$

It is true that 1 = 1. Only, we must be sure in every trial, that this condition is assured and respected and not, if at all, only in this case. In this single case, Bell's inequality is respecting classical logic, but not in general and at every trial. It is not up to Bell's inequality to decide if or if not to respect the law of the excluded middle. Bell's inequality must respect the law of the excluded middle at every trial. This is not the fact. Bell's inequality leads thus to logical contradictions and is not compatible with classical logic. Therefore, our original assumption, that Bell's theorem is correct is false.

Conclusion.

Bell's theorem contradicts classical logic and leads to a logical contradiction!
Bell's theorem is refuted!

Q. e. d.

There is more then an overwhelming evidence that Bell's is misleading and badly pronounced.

Bell's theorem - reductio ad absurdum IV.

Theorem 17. Bell's theorem contradicts classical logic and leads to a logical contradiction - reductio ad absurdum IV.

Premises.

Let

A_t	denote the random variable A_t which is either true (=1) or false (=0) at the (space) time t,
Not A_t	denote logical negation of the random variable A_t which is either true (=1) or false (=0) at the (space) time t,
C_t	denote the random variable C_t which is either true (=1) or false (=0) at the (space) time t,
Not C_t	denote logical negation of the random variable C_t which is either true (=1) or false (=0) at the (space) time t,
t	denote the (space)time t.

Let Bell's inequality be denoted by

$$\mathbf{(1 - ((1 - (A_t))^* (1 - (Not\ A_t)))) \geq (Not\ A_t) + (Not\ C_t) ^* ((A_t) - (B_t)).}$$

Assumption.

We want to prove that Bell's theorem is false. Thus, let us assume the opposite, the logical negation, of that what we want to proof. Let us assume that **Bell's theorem is correct**, that is to say true.

Proof by contradiction.

Eq.

$$(1 - ((1 - (A_t))^* (1 - (Not\ A_t)))) \geq ((Not\ A_t) + (Not\ C_t) ^* ((A_t) - (B_t))) \quad (96)$$

The term $((Not\ A_t) + (Not\ C_t) ^* ((A_t) - (B_t)))$ can take the values 0 or 1 according to Eq. (76) - (83). In so far, let us assume, that $\mathbf{((Not\ A_t) + (Not\ C_t) ^* ((A_t) - (B_t))) = 1}$. We obtain Eq. (97).

$$(1 - ((1 - (A_t))^* (1 - (Not\ A_t)))) \geq (((Not\ A_t) + (Not\ C_t) ^* ((A_t) - (B_t))) = \mathbf{1}). \quad (97)$$

It is generally accepted, that (a ≥ b) means that (a = b) **or** (a > b), both are equally allowed and possible, if the inequality is true. In so far, Eq. (96) is true, if $((1-((1-(A_t))^*(1-(Not\ A_t)))) = 0)$. Eq. (96) is equally true if $((1-((1-(A_t))^*(1-(Not\ A_t)))) > 0)$. In this case, let us assume, that

$$(1-((1-(A_t))^*(1-(Not\ A_t)))) > (((Not\ A_t) + (Not\ C_t) ^* ((A_t) - (B_t))) = 1),$$

which satisfies Bell's inequality. On the other hand, Bell is respecting classical logic and thus the law of the excluded middle as proofed in Eq. (75). The law of excluded middle in classical bivalent logic must yield $\mathbf{(1-((1-(A_t))^*(1-(Not\ A_t)))) = 1}$. Bell's inequality is respecting this law. We obtain Eq. (98).

$$((1 - ((1 - (A_t))^* (1 - (Not\ A_t)))) = \mathbf{1}) > (((Not\ A_t) + (Not\ C_t) ^* ((A_t) - (B_t))) = 1) \quad (98)$$

Eq. (98) is equal to Eq. (99).

$$\mathbf{1 > 1.} \quad (99)$$

It is not true that 1 > 1. Further, according to Bell's inequality, the law of the excluded middle is respected if **(1-((1-(A_t))*(1-(Not A_t)))) > 1**, which is not what classical logic demands. Classical logic demands that the law of the excluded middle is respected if **(1-((1-(A_t))*(1-(Not A_t))))=1.** Bell is claiming to respect classical logic, but in fact he does not. Therefore, Bell's inequality leads once again to a logical contradiction. Our original assumption, that Bell's theorem is correct, is false.

Conclusion.

Bell's theorem contradicts classical logic and leads to a logical contradiction!
Bell's theorem is refuted!

Q. e. d.

Bell's theorem - reductio ad absurdum V.

Example.

Some CIA Agents are prepared for a mission at the CIA Headquarter. Each Agent is either male of female, each agent has three closed letter named as D, S and L inside his pocket. Inside every letter there is a paper with a written secret which denotes our local hidden variables. The secret on the paper inside every letter can take only the following values: either "local hidden variable = 1" which denotes that this is a local hidden variable or "local hidden variable = 0" which denotes that this is not a local hidden variable, that is to say there is no local hidden variable inside the letter opened.

The CIA Agents with the properties D, S and L are sent from the CIA Headquarter in two opposite directions A and B while each of them using a car. Each car is stopped and controlled at the check-points A and B. Further, it is assured, that when a male agent is sent in direction A, a female agent is sent at the same time in direction B and vice versa.

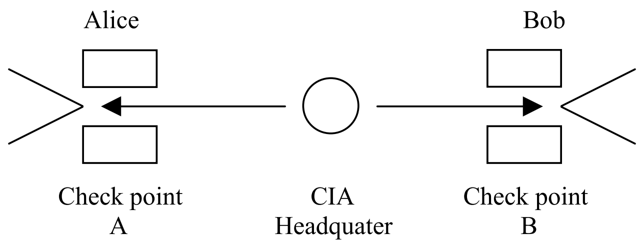

Figure. Test of Bell`s **thought** experiment.

At check-point A police officer Alice is doing her job. At the check-point B, police officer Bob is serving, both perform independent measurement and are recording the results. The data are the following.

Theorem 18. Bell's inequality - an Example.

Let

D_t	denote the random variable A_t which is either true (=1) or false (=0) at the (space) time t,
Not D_t	denote logical negation of the random variable A_t which is either true (=1) or false (=0) at the (space) time t,
S_t	denote the random variable A_t which is either true (=1) or false (=0) at the (space) time t,
Not S_t	denote logical negation of the random variable A_t which is either true (=1) or false (=0) at the (space) time t,
L_t	denote the random variable C_t which is either true (=1) or false (=0) at the (space) time t,
Not L_t	denote logical negation of the random variable C_t which is either true (=1) or false (=0) at the (space) time t,
t	denote the (space)time t.

In this context, Bell's inequality is denoted by

$$\left(D_t * \left(\mathrm{NotS}\right)_t\right) + \left(S_t * \left(\mathrm{NotL}\right)_t\right) \geq \left(D_t * \left(\mathrm{NotL}\right)_t\right).$$

Proof.

D_t	S_t	Not S_t	L_t (1)	Not L_t	(D_t* Not S_t)) (2)		(S_t* Not L_t)) (3)	(2) + (3)		(D_t* Not L_t))	**Bell's inequality**	**Eq.**
1	1	0	**1**	0	0	+	0	**0**	≥	**0**	not violated.	(100)
1	0	1	**1**	0	1	+	0	**1**	≥	**0**	not violated.	(101)
0	1	0	**1**	0	0	+	0	**0**	≥	**0**	not violated.	(102)
0	0	1	**1**	0	0	+	0	**0**	≥	**0**	not violated.	(103)
1	1	0	**0**	1	0	+	1	**1**	≥	**1**	not violated.	(104)
1	0	1	**0**	1	1	+	0	**1**	≥	**1**	not violated.	(105)
0	1	0	**0**	1	0	+	1	**1**	≥	**0**	not violated.	(106)
0	0	1	**0**	1	0	+	0	**0**	≥	**0**	not violated.	(107)
Q. e. d								**4**	≥	**2**	not violated.	

The situation doesn't change if we summarise the equation above as

$$\sum_{t=1}^{n}\left(D_t * \left(\mathrm{Not\ S}\right)_t\right) + \sum_{t=1}^{n}\left(S_t * \left(\mathrm{Not\ L}\right)_t\right) \geq \sum_{t=1}^{n}\left(D_t * \left(\mathrm{Not\ L}\right)_t\right). \qquad (108)$$

It is said that 1 ≥ 1 is true and that 0 ≥ 0 is true. In so far, the data of the experiment above overwhelmingly show that the inequalities of Bell's theorem are **not violated**. This provides dramatic empirical evidence if favour of local realism. However, have we done all right? We should have a more precisely look on the data above.

According to Eq. (100) - Eq. (103) **Bell's inequality is not violated** and equally there where **local hidden variables** (**L=1** in column (1)). Is so far it could be reasonable to say, Bell`s inequality is able to distinguish between local hidden variables and not local hidden variables. We should conclude, there are local hidden variable as there where indeed some. Only, there still remains a problem.

According to Eq. (104) - (107) there where **no local hidden variables** or L = 0 in column (1). Since we have assured that there are no local hidden variables, Bell's inequality must be violated in this case. But contrary to expectation, **Bell's inequality is not violated**. In so far, we arrived at a logical contradiction.

Bell's inequality is not violated if there are local hidden variables and equally Bell's inequality is not violated if there are no local hidden variables, this is a **contradiction**. In so far, this inequality is not able to distinguish between local hidden and local not hidden variables. The reason for this incapability is already identified at Eq. (75), it is the misuse of the law of the excluded middle and classical logic by Bell.

Let us recall once again that we started with Bell's inequality and assumed that the same is correct. It doesn't matter where we arrive, Bell's inequality leads to a logical contradiction and contradicts classical logic. At the end of our proofs we were able to derive logical contradictions from Bell's inequality. In so far, our proofs argues any relevance of Bell`s inequality. The law of the excluded middle, as one of the basic laws of classical logic, is not respected by Bell at all. Bell is not respecting classical logic, he is misusing the law of the excluded middle for his purposes, Bell has committed the fallacy of the excluded middle. As we said in another occasions (Barukčić, 2006b), Bell's theorem is a logical fallacy, Bell's theorem is a Black-and-White Fallacy, a false dilemma. It should be possible to derive a trilemma and a polylemma from Bell's inequality too. As a matter of fact, it is proofed that the law of the excluded middle defined in Bell's own way, is no longer the law of the excluded middle, it is something else, but not the law of the excluded middle. In so far, it is proofed that

Bell's theorem contradicts classical logic and

misuses the law of the excluded middle.

For that reason,

Bell's theorem is refuted, definitely!

4. Discussion

As proofed above, Bell's theorem is fallacious because of specifically logical reasons. The logic of Bell's theorem is not sound. Bell's theorem contradicts classical logic, it is based upon a fallacy. In so far either Bell's theorem is valid or classical logic is valid but not both. Bell's theorem is not compatible with the law of the excluded middle, it is a fallacy of the excluded middle. Bell has committed the fallacy of the excluded middle, commonly referred to as a false dilemma. This logical fallacy is sometimes known also as a false correlative, an either/or fallacy, a bifurcation or as black and white thinking. Bell's formalisation of local realism, his starting point, is incorrect and is based on a logical contradiction. Bell's theorem, as a false dilemma fallacy, refers to a misuse of the law of the excluded middle. Bell has misapplied the law of excluded middle at an maximum. An extreme simplification, a wishful thinking and a misapplication of the law of the excluded middle is the foundation of Bell's theorem.

Bell's theorem is the most profound logical fallacy of science.

Further, Bell's theorem is the definite and best proof known, that correlation analysis contradicts Quantum mechanics and Relativity Theory, that it is a useless and dangerous statistical machinery. In so far, as proofed above, Bell's theorem is definitely refuted. The book on Bell's theorem is completely closed.

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